



# **NAVAL POSTGRADUATE SCHOOL**

**MONTEREY, CALIFORNIA**

## **THESIS**

**VISION BASED CONTROL AND TARGET RANGE  
ESTIMATION FOR SMALL UNMANNED AERIAL VEHICLE**

by

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December 2005

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**VISION BASED CONTROL AND TARGET RANGE ESTIMATION FOR  
SMALL UNMANNED AERIAL VEHICLE**

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## **ABSTRACT**

In the tracking of a moving ground target by small unmanned air vehicle (UAV) via camera vision, the target position and motion cannot be measured directly. Two different types of filters were assessed for their ability to estimate target motion, namely target velocity, directional heading on flat ground and distance from the UAV to target. The first filter is a nonlinear deterministic filter with stability guarantee. The second filter is based on nonlinear Kalman Filter technique. The application and performance of these two filters are presented, for simulated vision based target tracking.

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## LIST OF ABBREVIATIONS, ACRONYMS, SYMBOLS

### ACRONYMS

|      |                               |
|------|-------------------------------|
| SUAV | Small Unmanned Aerial Vehicle |
| UAV  | Unmanned Aerial Vehicle       |
| LOS  | Line of Sight                 |
| DOF  | Degree of Freedom             |
| GPS  | Global Positioning System     |
| NPS  | Naval Postgraduate School     |
| LPV  | Linear Parametrically Varying |
| TNT  | Tactical Network Topology     |

### SYMBOLS

|           |   |
|-----------|---|
| $\phi$    | Roll angle  |
| $\theta$  | Pitch angle   |
| $\varphi$ | Yaw angle   |
| $\eta$    | Angle from vector perpendicular to line of sight, to velocity vector      |
| $\lambda$ | Angle between line of sight vector to target and x-axis of inertial frame |
| $\rho$    | Range from UAV to target  |
| $\psi$    | Directional heading, measured from x-axis of inertial frame               |
| $V$       | Velocity vector   |
| $p$       | Position vector   |

### SUPERSCRIPTS AND SUBSCRIPTS

|   |                 |
|---|-----------------|
| I | Inertial frame  |
| B | Body frame      |
| C | Camera frame    |
| t | Target notation |
| g | SUAV notation   |

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# **I. INTRODUCTION**

## **A. BACKGROUND**

The goal of this project was to enhance the target tracking features of the airborne sensor in support of the Tactical Network Topology (TNT) experiment, in which the Naval Postgraduate School (NPS) is participating. This experiment assesses the information flow in a network through scenario plays and gathers part of the required information through various sensors. The airborne sensor is one such sensor through which ground target information can be collected. Currently the airborne sensor includes the Small Unmanned Aerial Vehicle (SUAV) equipped with a pan-tilt camera for target tracking purpose. This sensor was previously developed in NPS as a system that incorporates ground target tracking control and SUAV guidance. The SUAV to target distance information was used to guide the SUAV to fly in a circular path, to facilitate continuous tracking by its onboard camera. The current target tracking process focuses on a stationary ground target and is able to estimate the range from the SUAV to the target. In the case of a moving ground target, the current tracking process does not yield information on the speed and direction which the target is traveling.

## **B. PROBLEM FORMULATION**

The purpose of this project was to investigate the use of a filter to estimate the ground target speed and heading. In this thesis, the applications of two different filters were discussed, with regards to the formulation of the filter and also the filter performance in tracking motion.

In order to assess the filter performance, existing SUAV truth models were used to provide flight and camera models. The main focus of this project was the estimation of target speed and the range from SUAV to target. The range is an important variable to be estimated, as this has a bearing on the guidance for the SUAV flight pattern.

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## II. LITERATURE SURVEY

### A. RANGE ESTIMATION FOR STATIONARY TARGET

The study and design of the control system using SUAV and onboard vision device for tracking stationary target was previously carried out in NPS. In this section that follows, the technique of range estimation by Prince [Ref 1] is summarized.

The range from the SUAV to a ground target was estimated using linear discrete Kalman Filter. The range was subsequently used to guide the SUAV to fly in a circular path around the target, so that the SUAV can maintain a defined distance from the target during the tracking process. The Kalman filtering technique was employed to estimate the range from SUAV to target and the filter's system equation was given by

$$x_{k+1} = Fx_k + w_k \quad [\text{Ref 1}]$$

where  $x_k = \begin{bmatrix} \rho_k \\ \dot{\rho}_k \end{bmatrix}_k$ ,  $F = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$  and  $w_k \sim N(0, Q_k)$  is the process noise with covariance  $Q_k$ ,  $\Delta t$  denotes the sample time and  $\rho$  denotes the range from SUAV to the target. The measurement equation was given by

$$z_k = Hx_k + v_k \quad [\text{Ref 1}]$$

where  $z_k = \begin{bmatrix} V_k^P \\ \dot{\rho}_k \end{bmatrix}_k$ ,  $H_k = \begin{bmatrix} \hat{\lambda}_k & 0 \\ 0 & 1 \end{bmatrix}$  and  $v_k \sim N(0, R_k)$  is the measurement noise

with covariance  $R_k$ ,  $\hat{\lambda}_k$  denotes the estimated line-of-sight (LOS) rate and  $V_k^P$  denotes the SUAV velocity vector that is perpendicular to, the LOS to target, where  $V_k^P = \rho \dot{\lambda}$ . The estimated line-of-sight rate  $\hat{\lambda}_k$  was obtained using another set of system and measurement equations, see Prince [Ref 1] for more details.

Constant Kalman gains  $K_{k+1}$  could be used in the measurement update equation to obtain the updated range estimate. The measurement update was given by

$$\hat{x}_{k+1} = \bar{x}_{k+1} + K_{k+1}(z_{k+1} - H_{k+1}\bar{x}_{k+1}) \quad [\text{Ref 1}]$$

This application of Kalman filter to this problem was successful in obtaining the estimated range thus enabling the use of range information for SUAV flight path control and for determining target location.

Another approach to estimate the range to a stationary target was done by Wang et al [Ref 2] using nonlinear Kalman filtering. The following diagram shows the definitions of key variables used in the tracking kinematics.

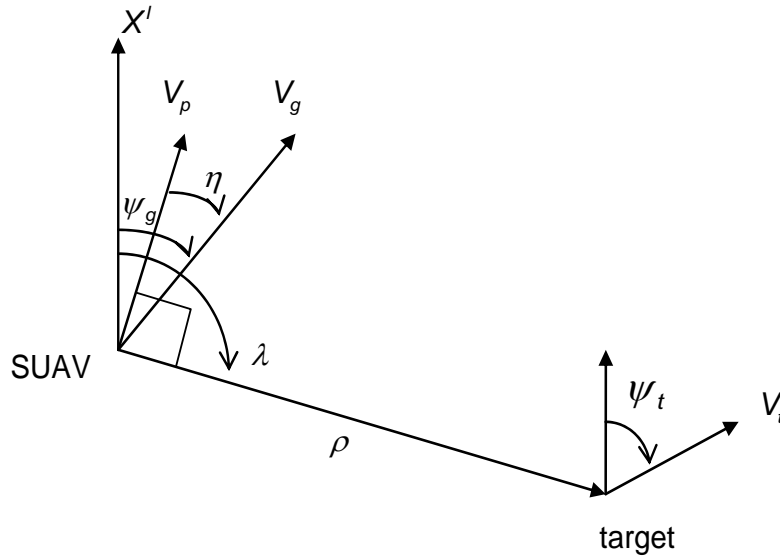


Figure 1. Moving target tracking in inertial  $X_I$  and  $Y_I$  frame [After: Ref 2].

The process model was given by

$$\begin{aligned} \dot{x} &= f(x, \psi) \\ z &= h(x) \end{aligned} \quad [\text{Ref 2}]$$

$$\text{where } x = \begin{bmatrix} \eta \\ \rho \\ \dot{\lambda} \end{bmatrix}, f(x, \dot{\psi}) = \begin{bmatrix} -\dot{\lambda} + \dot{\psi} \\ -V_g \sin \eta \\ \frac{2V_g \sin \eta}{\rho} \dot{\lambda} - \frac{V_g \sin \eta}{\rho} \dot{\psi} \end{bmatrix},$$

$$z = \begin{bmatrix} \eta \\ V_\rho \end{bmatrix} = \begin{bmatrix} \eta \\ V_g \cos \eta \end{bmatrix} \text{ and } h(x) = \begin{bmatrix} \eta \\ \rho \dot{\lambda} \end{bmatrix}. \quad [\text{Ref 2}]$$

A series of steady state Kalman gains  $K$  was computed based on several range  $\rho$  and estimation of the range to the moving target was obtained in simulation. In the same study, the theoretical range was obtained through derivation of kinematics relationship between SUAV and target.

## B. VELOCITY ESTIMATION OF UNDERWATER VEHICLE

In separate study by Oliveira et al [Ref 3], an Autonomous Surface Craft (ASC) tracked the velocity of an Autonomous Underwater Vehicle (AUV) and estimated the velocity of the later using a nonlinear estimator. The nonlinear relationship of this tracking problem was solved based on the theory of linear parametrically varying system. This section describes the study by Oliveira et al [Ref 3].

There are two parts to the solution, a process model and a tracker design. The process model comes in the form:

$$\begin{aligned} \dot{p} &= -{}^I_S R(\lambda) \cdot {}^I v_s)_m + b + w_v \\ \dot{b} &= 0 \\ y_m &= h_\lambda({}^C p) + w_y \end{aligned} \quad [\text{Ref 3}]$$

where  $p$  was the position of the AUV,  $b$  was the velocity bias to be estimated. The rotation matrix transforming  $\{S\}$  to  $\{I\}$  frame is  ${}^I_S R(\lambda)$  and  $w$  is the noise input. The measurement  $y_m$  is given by  $y = [u_c \quad v_c \quad z]^T$ ,  $h_\lambda({}^C p)$  was the mapping of the position of AUV, with respect to camera frame, into the camera image plane  $u_c$ ,  $v_c$  and vertical height  $z$  from ASC to AUV. The figure below shows the relationship of ASC/camera and AUV, in relation to the inertial frame.

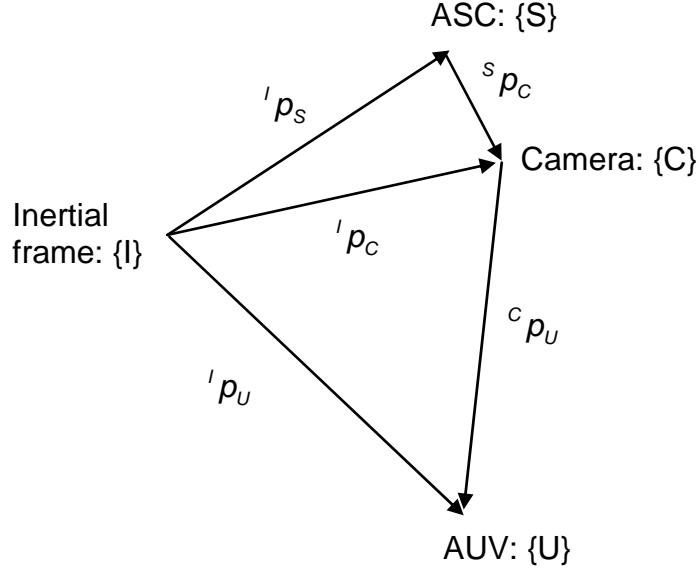


Figure 2. Tracking of underwater AUV target [After: Ref 3].

The key to estimating the AUV velocity is the relationship:

$$h_{\lambda}({}^C\hat{p}) - h_{\lambda}({}^Cp) = L({}^C\hat{p}, {}^Cp)H({}^C\hat{p})({}^C\hat{p} - {}^Cp) \quad [\text{Ref 3}]$$

where  $L({}^C\hat{p}, {}^Cp) = \text{diag}(\hat{z}_C / z_C, \hat{z}_C / z_C, 1)$  and  $H({}^C\hat{p})$  is the Jacobian of  $h_{\lambda}({}^C\hat{p})$ . By having  $L({}^C\hat{p}, {}^Cp) = I$  if  $\hat{z}_C / z_C \approx 1$ , the expression becomes

$$({}^C\hat{p} - {}^Cp) = H({}^C\hat{p})^{-1} \cdot (h_{\lambda}({}^C\hat{p}) - h_{\lambda}({}^Cp)) \quad [\text{Ref 3}] \quad (1)$$

This expression relates errors in sensor measurement to errors in the estimation variables, thus casting the estimation problem in linear parametrically varying system (LPV) framework. The filter realization is given by:

$$\begin{aligned} \dot{\hat{p}} &= -{}_S^I R(\lambda)^S ({}^I v_S)_m + \hat{b} + K_1 \cdot {}_C^I R(\lambda) H^{-1}({}^C\hat{p}) (h_{\lambda}({}^C\hat{p}) - y_m) \\ \dot{\hat{b}} &= K_2 \cdot {}_C^I R(\lambda) H^{-1}({}^C\hat{p}) (h_{\lambda}({}^C\hat{p}) - y_m) \end{aligned} \quad [\text{Ref 3}]$$

This filter realization is also shown in figure 3.

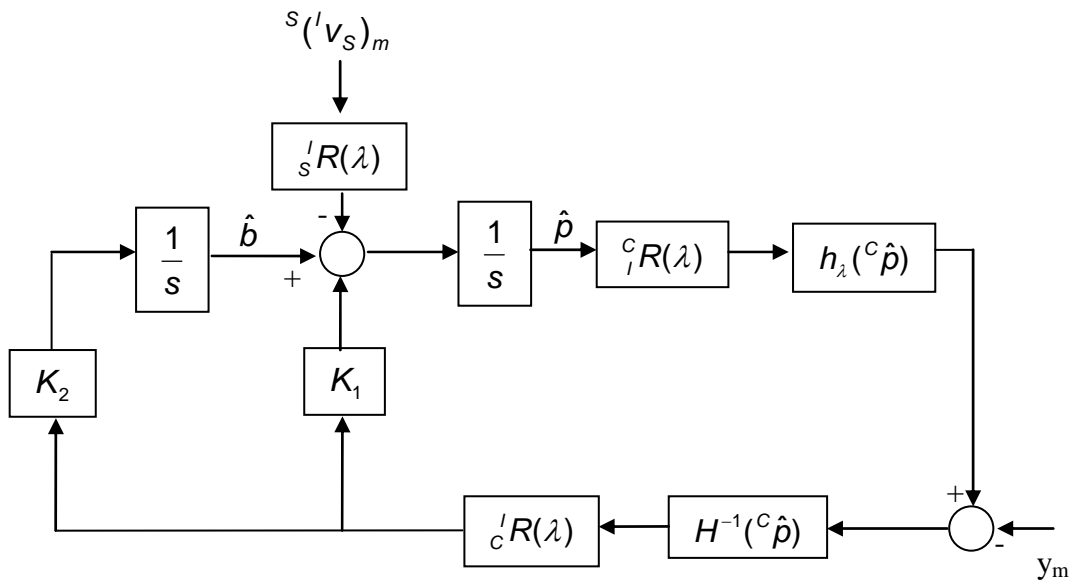


Figure 3. Tracker structure [After: Ref 3].

Estimator gains  $K_1$  and  $K_2$  were selected to achieve desired performance for the filter.

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### III. PROBLEM FORMULATION

#### A. PROBLEM FORMULATION

In the subsequent chapters, two approaches to estimation of range and velocity of the moving ground target are described. The first approach uses the filter based on LPV system as described in chapter II. The second approach attempts to estimate the target range and velocity based on continuous nonlinear Kalman filtering using steady state Kalman gains. Both approaches will be assessed through simulations using the MATLAB tool.

##### 1. Nonlinear Deterministic Filter with Stability Guarantee

The following diagram shows the framework for SUAV and target in the inertial frame, which will be used in the subsequent velocity estimation.

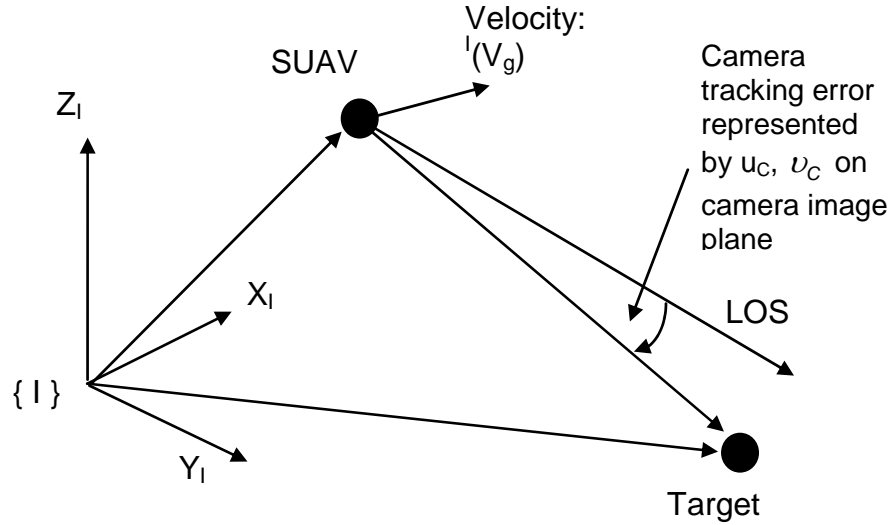


Figure 4. SUAV tracking a moving target in inertial frame.

The approach to estimate target range and velocity requires measurement inputs from the tracking errors  $u_c$  and  $v_c$ , as well as altitude  $z$  and SUAV velocity. Altitude was assumed to be obtained from the SUAV altimeter. The velocity was assumed to be obtainable from GPS based computation.

For the purpose of this study, an already existing UAV truth model was used to provide the required inputs to the filter. The filter was assessed for its ability to track the target with and without the addition of measurement noise.

## 2. Filter Based on Kalman Filtering Technique

The following diagram shows the framework for SUAV and target motion in the inertial frame. Here the framework is two dimensional as compared to the previous filter approach which was three dimensional.

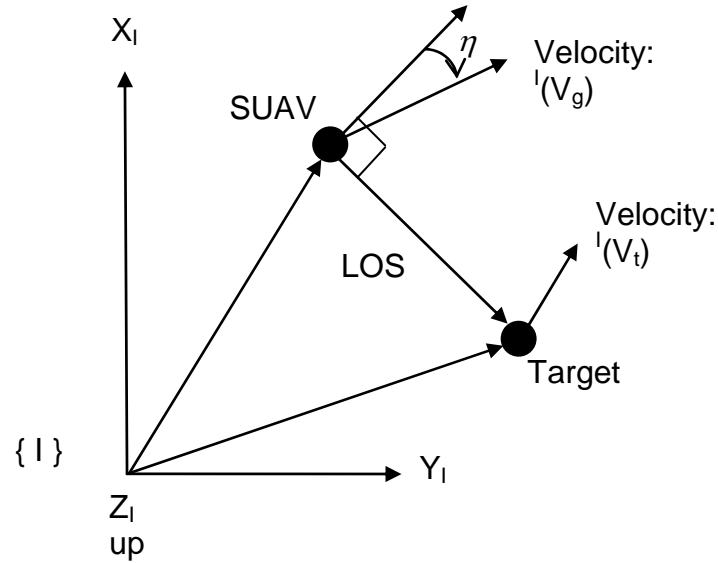


Figure 5. Target tracking framework for Kalman filter approach.

The Kalman filter approach required collection of measurements for velocity  $V_g$  and angle  $\eta$ . These measurements were compared with the estimated  $\hat{V}_g$  and  $\hat{\eta}$ . Angle  $\eta$  is the angle between the vector  $V_p$ , which is perpendicular to the LOS, and the velocity vector  $V_g$ . These values were assumed to be obtainable from the SUAV flight data. An already existing UAV truth model was used to provide the necessary inputs to the filter. Together with the process model, which will be discussed in more detail, the computed Kalman gain was used to generate new estimates of the state variables.

## B. COORDINATE SYSTEM

### 1. Camera Coordinates

The camera frame is denoted by  $\{C\}$  and has coordinates  $X_c$ ,  $Y_c$  and  $Z_c$  where  $X_c$  is the distance to the target and has its origin at the camera pin-hole location.  $X_c$  is positive in the direction of the target, along the camera to target axis.  $Y_c$  is the lateral distance of the target from the  $X_c$  axis and  $Z_c$  is the vertical distance of the target, positive when pointing downwards from the  $X_c$  axis. The  $Y_c$  and  $Z_c$  position of the target will be represented as the  $u_c$  and  $v_c$  offset distance from the  $X_c$  axis respectively, on the camera image plane. The focal length  $f$  of the lens is 12mm. Finally the camera is located at a height  $Z$  from the target, in the inertial frame. The camera coordinate system is illustrated below.

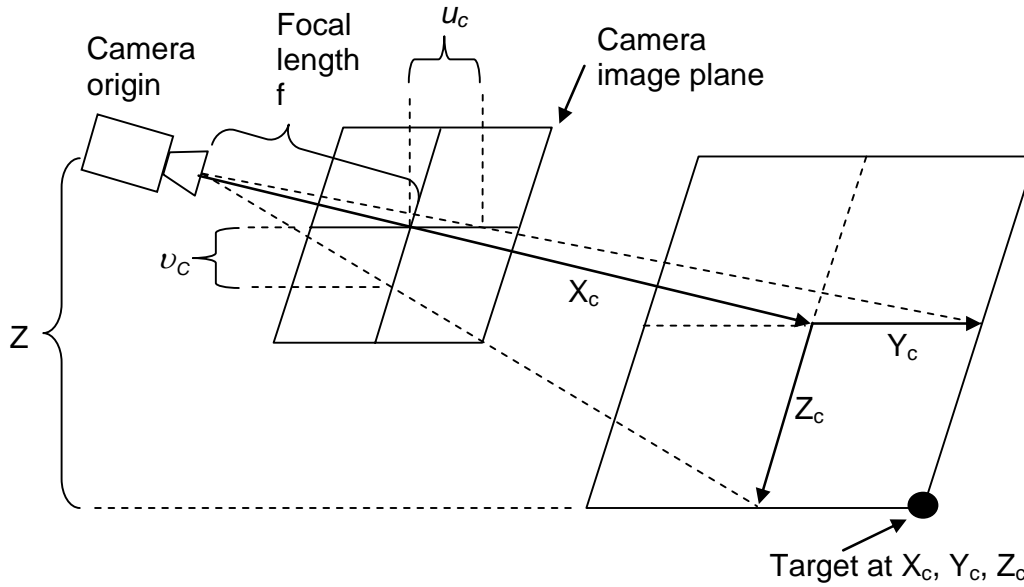


Figure 6. Camera coordinate system.

### 2. Gimbal Coordinates

The camera pointing angles or the gimbal angles are denoted by two angles namely gimbal pitch  $\theta_c$  (or tilt angle) and yaw  $\varphi_c$  (or pan angle). These are angled with respect to the SUAV airframe body coordinate system.

### 3. Body Coordinates

The SUAV airframe body frame is denoted by  $\{B\}$  and has coordinates  $X_B$  pointing towards the nose of the SUAV,  $Y_B$  pointing to right wing of SUAV and  $Z_B$  pointing upwards from the SUAV. The airframe is positioned in the inertial frame and rotated by the angles roll  $\phi_B$ , pitch  $\theta_B$  and yaw  $\varphi_B$ . These are computed with respect to the inertial frame.

### 4. Inertial Coordinates

The inertial coordinate system is denoted by  $\{I\}$  and has coordinates  $X_I$ ,  $Y_I$  and  $Z_I$ .

### 5. Transformation Matrix

The rotation matrix from camera to body frame [Ref 5] is:

$${}^B_c R = \begin{bmatrix} \cos \theta_c \cos \varphi_c & \cos \theta_c \sin \varphi_c & -\sin \theta_c \\ -\sin \varphi_c & \cos \varphi_c & 0 \\ \sin \varphi_c & \sin \varphi_c \sin \theta_c & \cos \theta_c \end{bmatrix}$$

The rotation matrix from body to inertial frame [Ref 5] is:

$${}^I_B R = \begin{bmatrix} \cos \theta_B \cos \varphi_B & \cos \theta_B \sin \varphi_B & -\sin \theta_B \\ \cos \varphi_B \sin \theta_B \sin \phi_B - \sin \varphi_B \cos \phi_B & \cos \varphi_B \cos \phi_B + \sin \varphi_B \sin \theta_B \sin \phi_B & \cos \theta_B \sin \phi_B \\ \cos \varphi_B \sin \theta_B \sin \phi_B + \sin \varphi_B \cos \phi_B & \sin \varphi_B \sin \theta_B \cos \phi_B - \cos \varphi_B \sin \phi_B & \cos \theta_B \cos \phi_B \end{bmatrix}$$

Then, the rotation matrices from camera to inertial frame and vice-versa are:

$${}^I_c R = {}^I_B R \cdot {}^B_c R \quad \text{and}$$

$${}^c_I R = ({}^I_c R)^T \quad \text{respectively.}$$

## IV. APPLICATION OF NONLINEAR DETERMINISTIC FILTER WITH STABILITY GUARANTEE

### A. DESCRIPTION OF FILTER

The filter described in Oliveira et al [Ref 3] was applied to the moving ground target tracking problem in this project. Instead of tracking the AUV, the filter technique was used to track the ground target. A six DOF SUAV truth model from Lizarraga [Ref 8] was used to generate required inputs for the filter. The following sections describe the filter in more detail.

#### 1. Process Model

The following process model in equations (2) to (4) [Ref 3] was used, in absence of noise, to implement the SUAV to ground target tracking:

$$\dot{p} = -({}^I v_g)_m + b \quad (2)$$

$$\dot{b} = 0 \quad (3)$$

$$y_m = h_\lambda({}^C p) \quad (4)$$

In equation (1) above, the first term  $-({}^I v_g)_m$  refers to the inertial speed of SUAV. The second term  $b$  denotes the actual target velocity which the filter will estimate. Hence the estimated velocity of the target  $\hat{V}_t$  will be based on  $\hat{b}$ . Based on the assumption that the target is traveling at a constant speed and heading, the time derivative  $\dot{b}$  is zero in equation (3). The first term in the measurement equation (4) converts the position of target in camera coordinates  ${}^C p$ , i.e.,  $x_c$ ,  $y_c$  and  $z_c$  into the image plane coordinates and altitude difference  $z_l$ , according to the relationship:

$$y = \begin{bmatrix} u_c \\ v_c \\ z_l \end{bmatrix} = \begin{bmatrix} \frac{f \cdot y_c}{x_c} \\ \frac{f \cdot z_c}{x_c} \\ R_{13} \cdot x_c + R_{23} \cdot y_c + R_{33} \cdot z_c \end{bmatrix} \quad \text{[Ref 3]} \quad (5)$$

where  $f$  is 12mm,  $R_{12}$ ,  $R_{23}$  and  $R_{33}$  are the elements of the third column of the rotation matrix  ${}^c_iR$ . Equation (5) represents the measurement equation  $y_m = h_\lambda({}^c p)$ .

The process model was then used to design the filter following Oliveira et al [Ref 3] with minor modification:

$$\dot{\hat{p}} = -({}^l v_g)_m + \hat{b} + K_1 \cdot {}^l_c R \cdot H^{-1}({}^c \hat{p})(h_\lambda({}^c \hat{p}) - y_m) \quad (6)$$

$$\dot{\hat{b}} = K_2 \cdot {}^l_c R \cdot H^{-1}({}^c \hat{p})(h_\lambda({}^c \hat{p}) - y_m) \quad (7)$$

The notation  $H({}^c \hat{p})$  refers to the Jacobian of  $h_\lambda({}^c \hat{p})$  and  $H^{-1}({}^c \hat{p})$  is the inverse of  $H({}^c \hat{p})$ . The selection of gains  $K_1$  and  $K_2$  will be described in the next section. The resulting implementation is shown below:

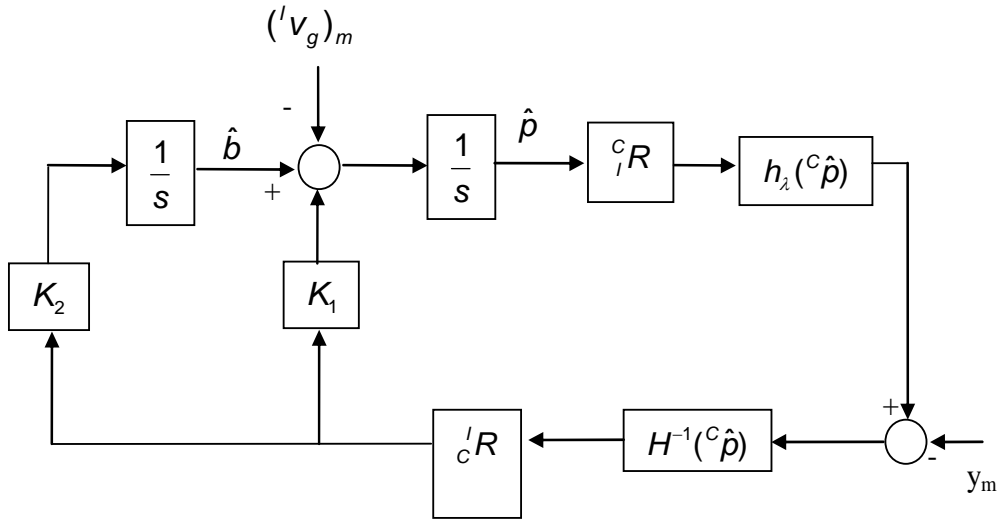


Figure 7. Tracker structure [After: Ref 3].

## 2. Gain Selection

Using identity (1) and assuming  $\hat{x}_c = x_c$ , the filter dynamics are given by

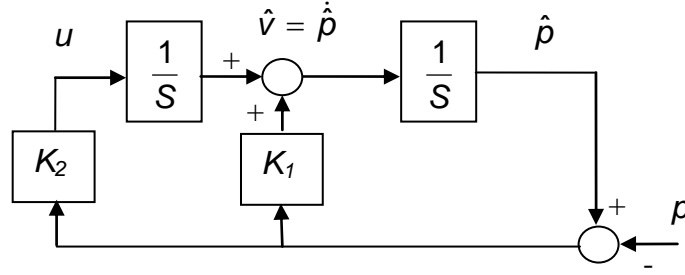


Figure 8. Basic position model.

By implementing the observer according to Ogata [Ref 7], which has the form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y_m - C\hat{x}), \text{ where } A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix}, C = [I \ 0], L = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \text{ with}$$

$K_1$  and  $K_2$  being the gains shown in the filter structure,  $K_1$  and  $K_2$  can be found by pole placement technique as described in Ogata [Ref 7]. The poles selected were  $[-3 \ -3 \ -3 \ -1 \ -1 \ -1]$  and the resulting gains were:

$$K_1 = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix} \text{ and } K_2 = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

These were the initial gains used to assess the filter performance. The gains were subsequently varied to address noise.

## B. RESULTS AND DISCUSSION

### 1. Filter Performance in Absence of Noise

The following graphs show the performance of the filter when the SUAV truth model was configured such that the SUAV attempts to circle around a moving target traveling at velocity  $[10, 5, 0]$  m/s in the inertial frame, along  $x_i$ ,  $y_i$  and  $z_i$  axis.

The results showed that the convergence of estimated target velocity was achieved in five seconds in absence of measurement noise.

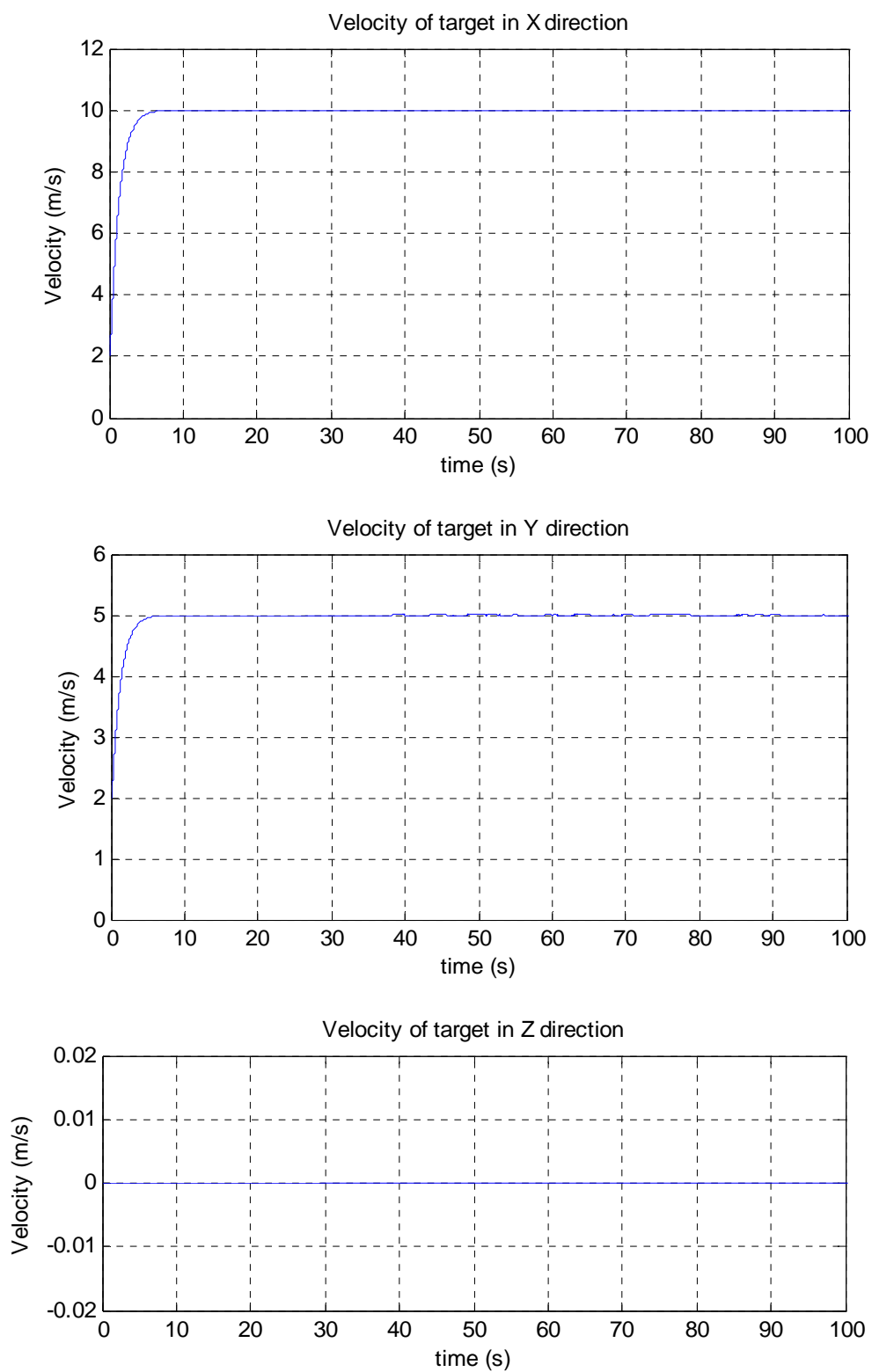


Figure 9. Estimated target velocity in inertial frame.



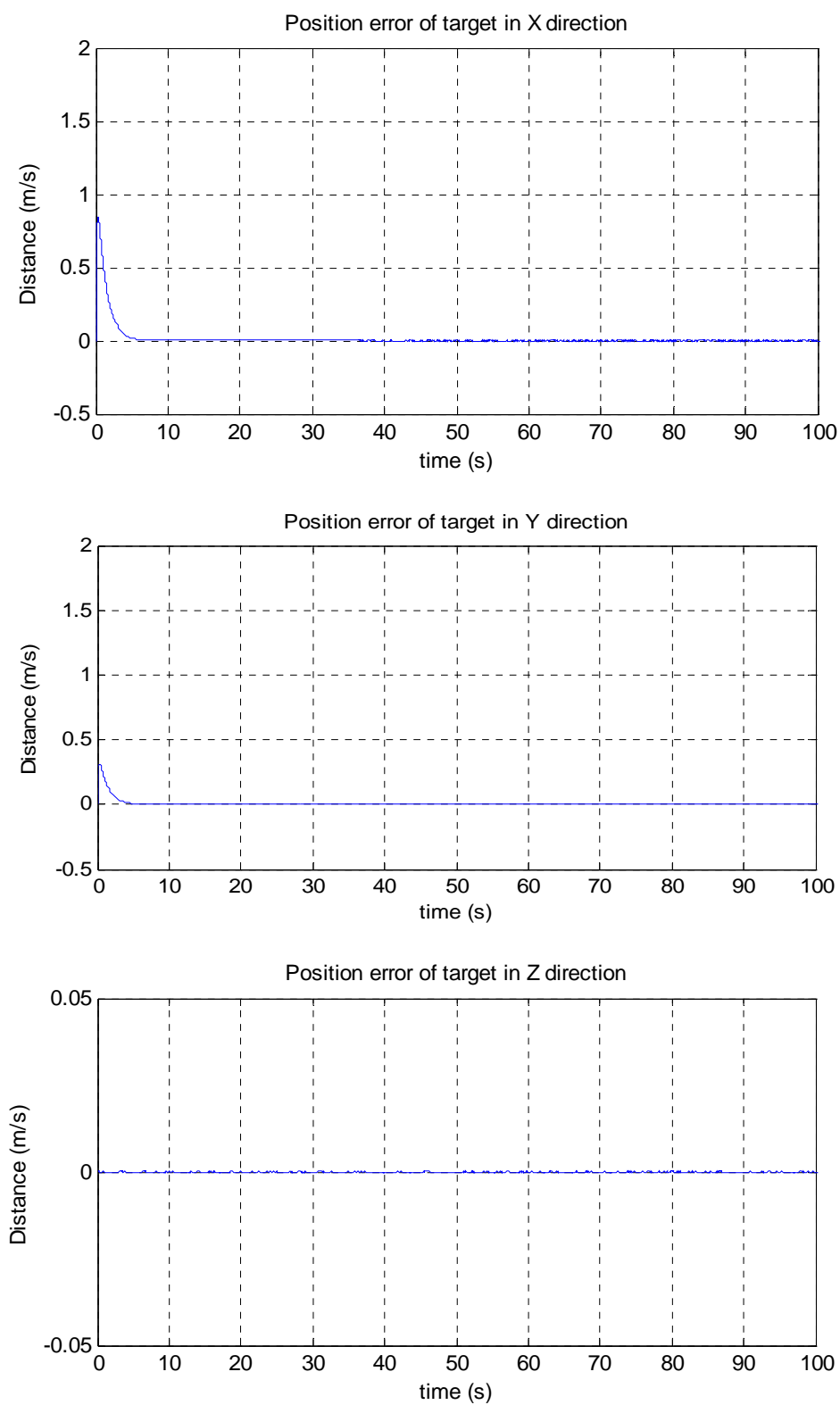


Figure 10. Target position error in inertial frame.

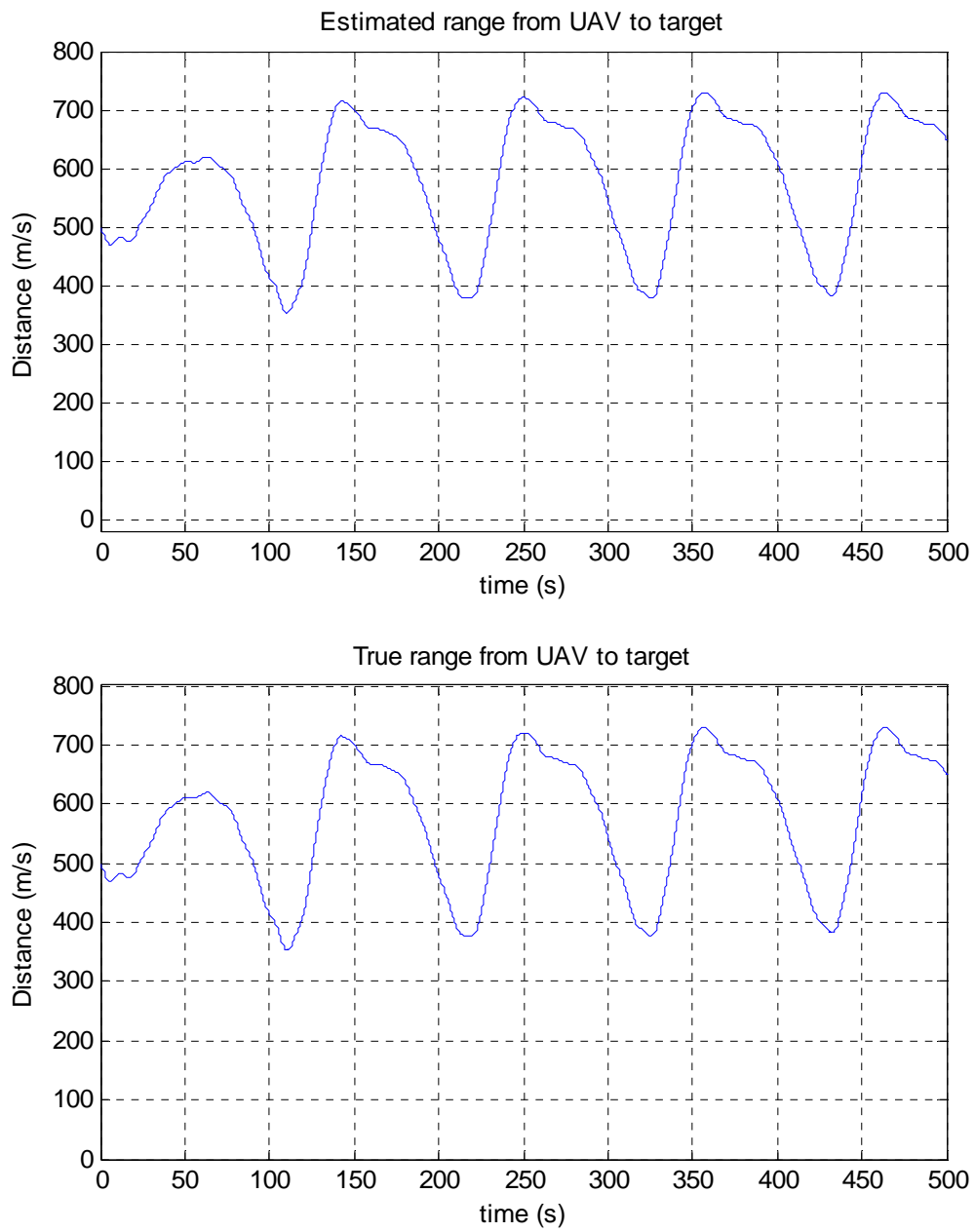


Figure 11. Range from SUAV to target.

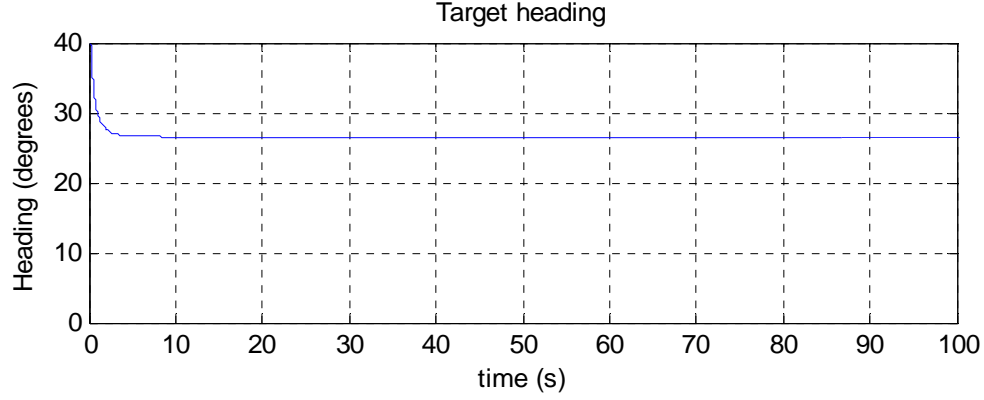


Figure 12. Estimated target heading.

The target heading was computed from the angle resulting from the vector summation of the estimated target velocity components along  $x_1$  and  $y_1$  axis. The estimated target heading corresponded well with the true target heading of 26.6 degrees.

By reducing the gains  $K_1$  and  $K_2$ , it was observed that the convergence for the estimated state variables were slower. In the example involving velocity, the convergence was around 60s. The gains as a result of pole selection of  $[-0.3 -0.3 -0.3 -0.1 -0.1 -0.1]$ , were as follows.

$$K_1 = \begin{bmatrix} -0.4 & 0 & 0 \\ 0 & -0.4 & 0 \\ 0 & 0 & -0.4 \end{bmatrix} \text{ and } K_2 = \begin{bmatrix} -0.03 & 0 & 0 \\ 0 & -0.03 & 0 \\ 0 & 0 & -0.03 \end{bmatrix}$$

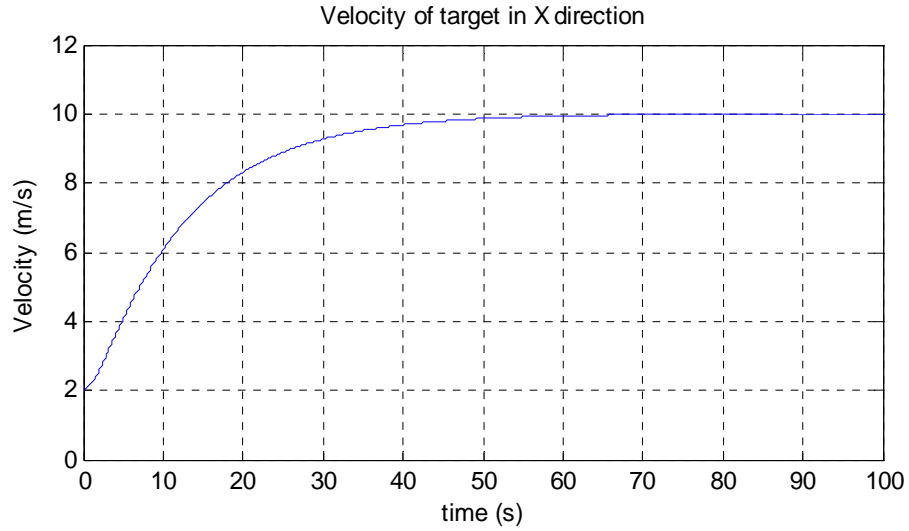


Figure 13. Example of velocity estimation with lower gains.

## 2. Filter Performance with Noisy Measurements

The following graphs showed the estimation of target motion when zero mean white noise was added to the measurements. The rms in velocity channel was  $\pm 2$  m/s, camera pan/tilt rms was  $\pm 0.3$  degrees, SUAV euler angles rms was  $\pm 2.8$  degrees, image plane error  $u_C$  &  $v_C$  rms was  $\pm 5$  degrees, and height  $z_I$  rms from SUAV to target was  $\pm 9$  m.

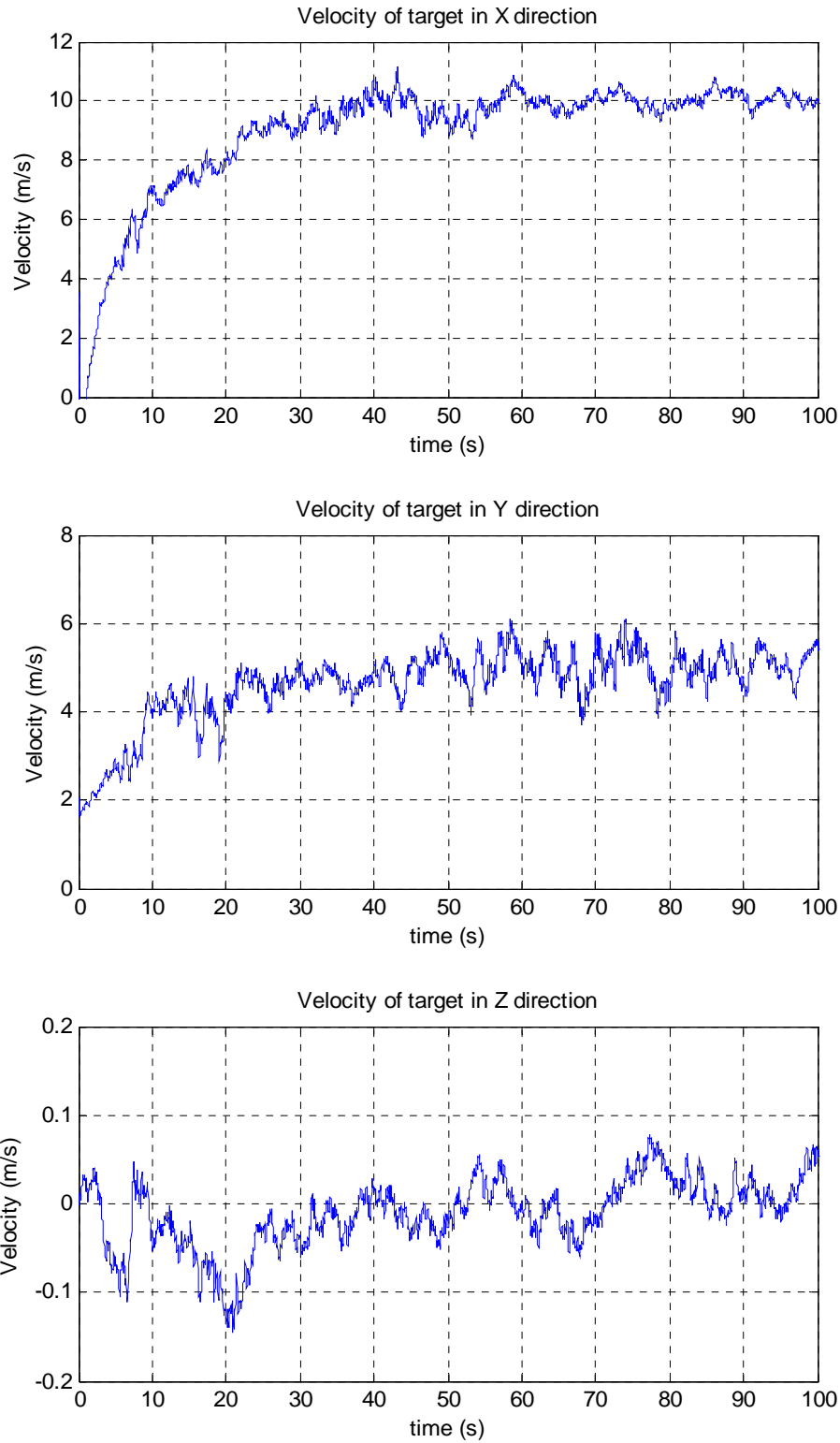


Figure 14. Estimated velocity in inertial frame (with measurement noise).

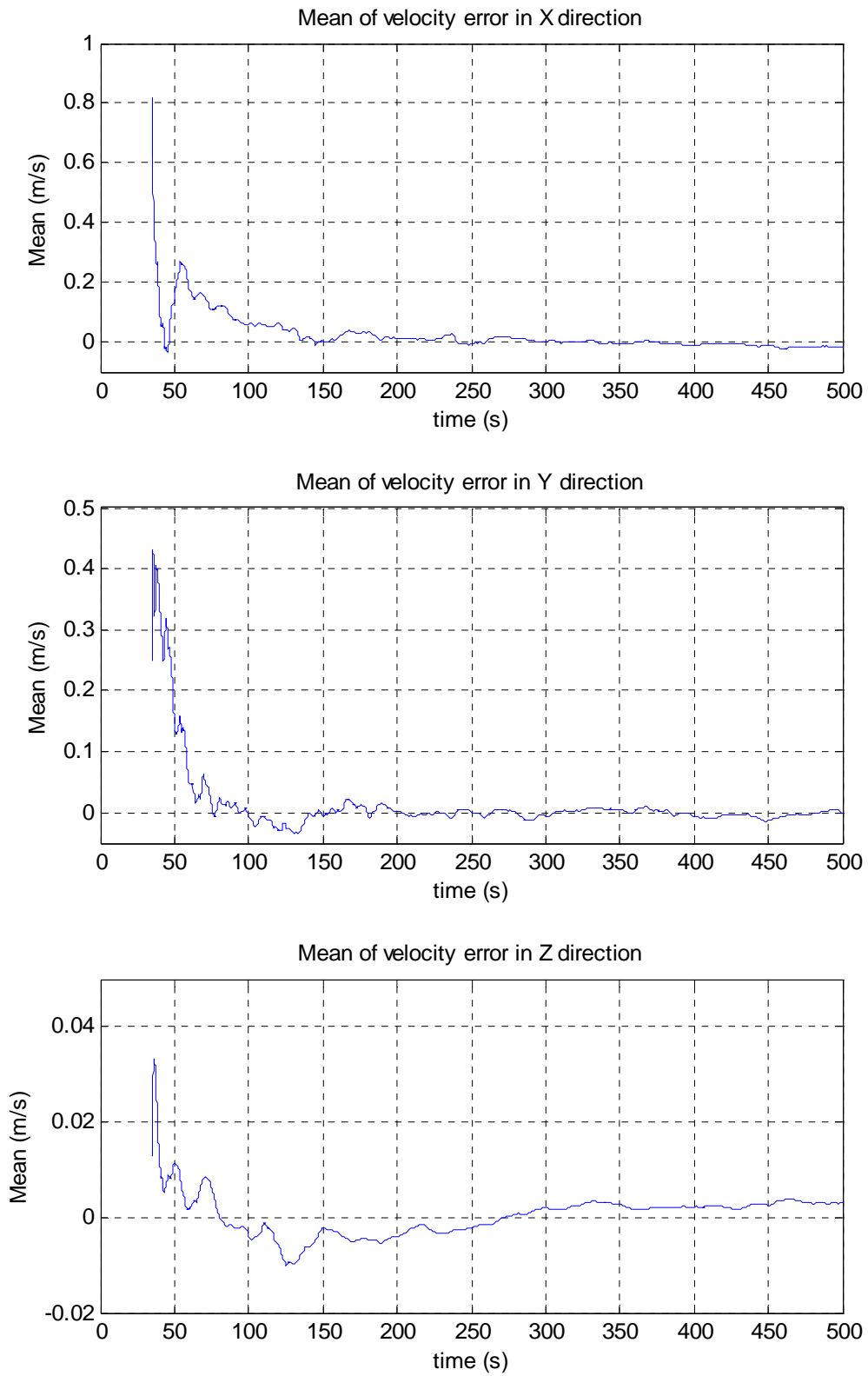


Figure 15. Mean of target velocity error.

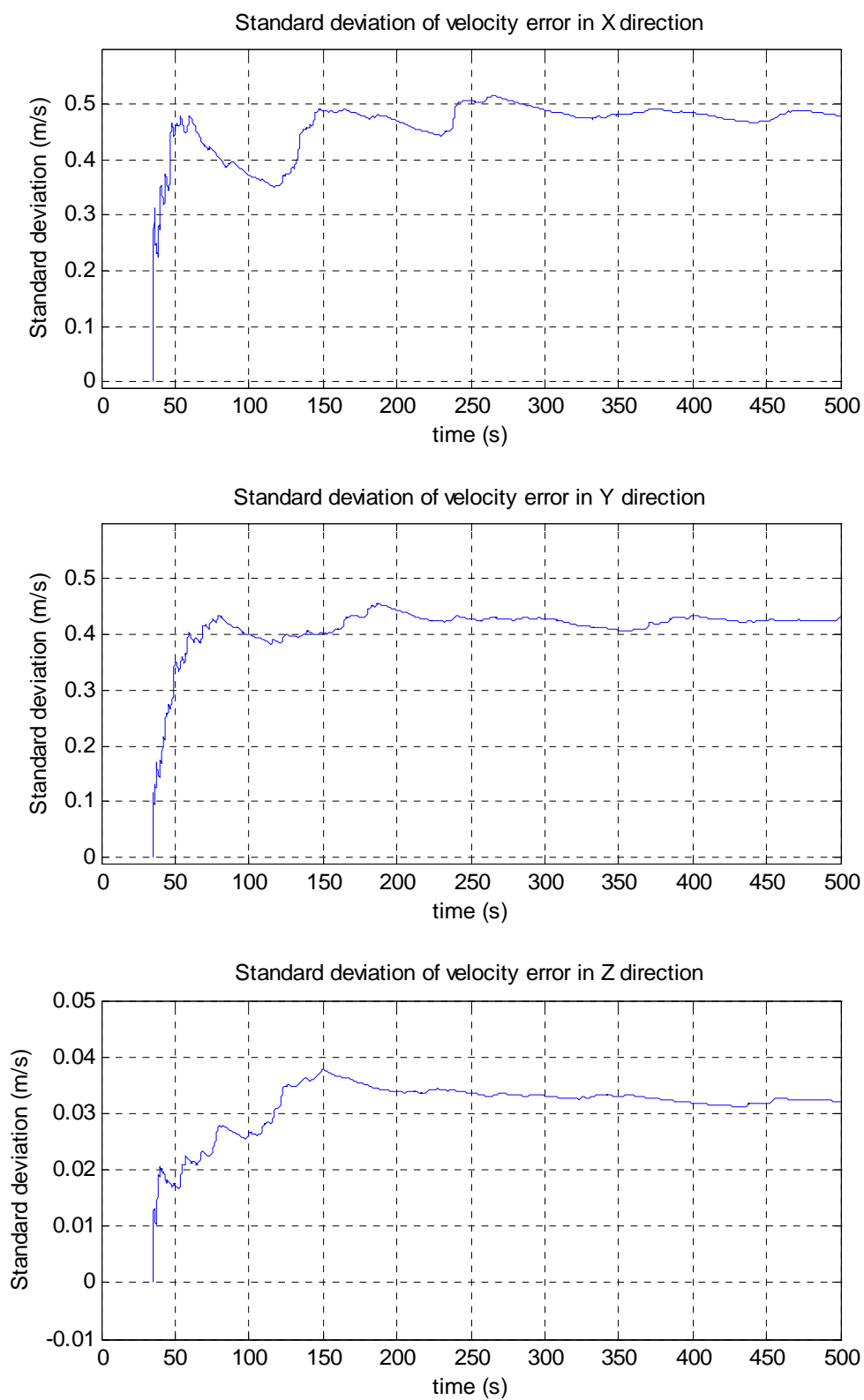


Figure 16. Standard deviation of target velocity error.

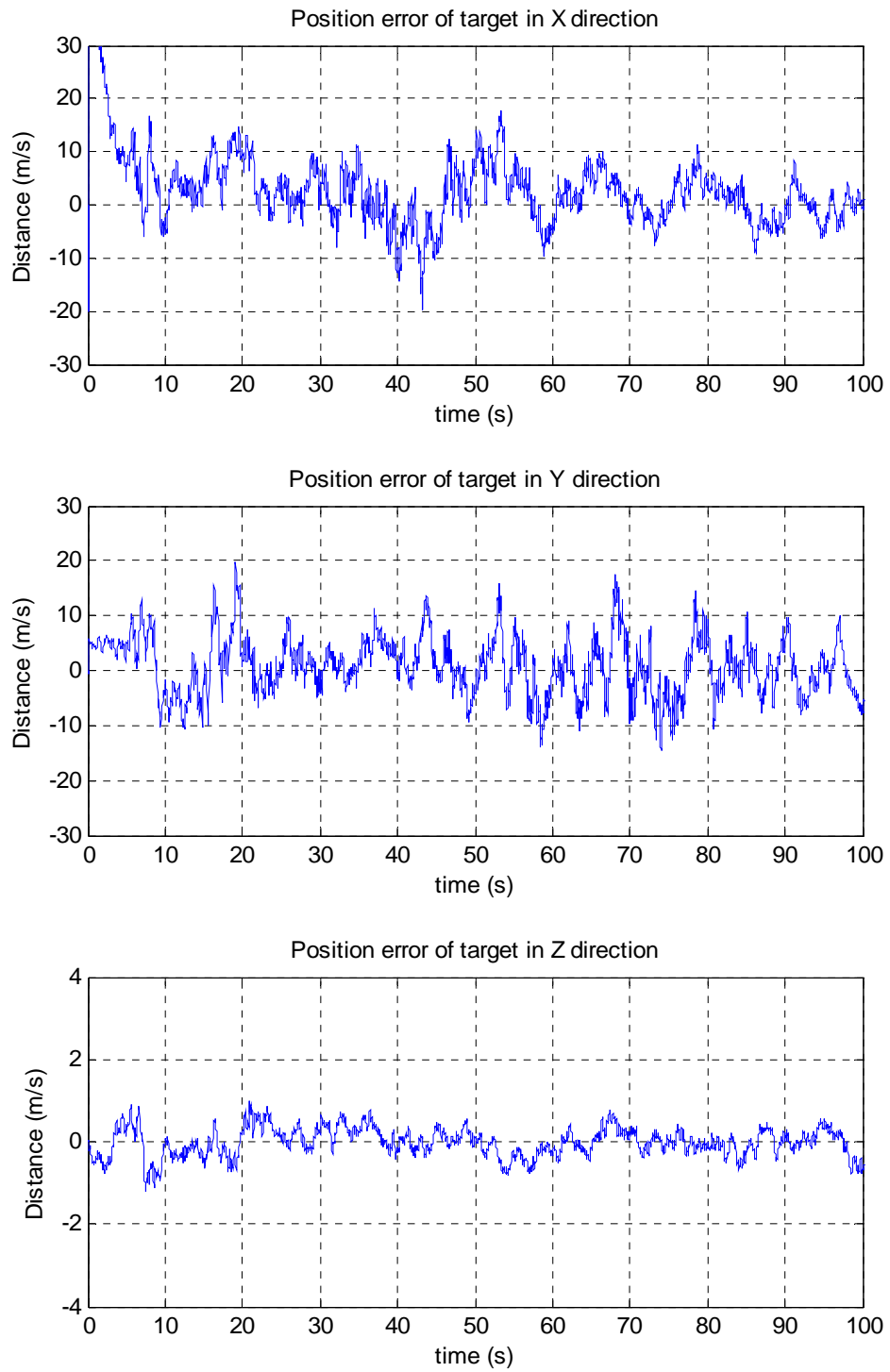


Figure 17. Target position error (with measurement noise).



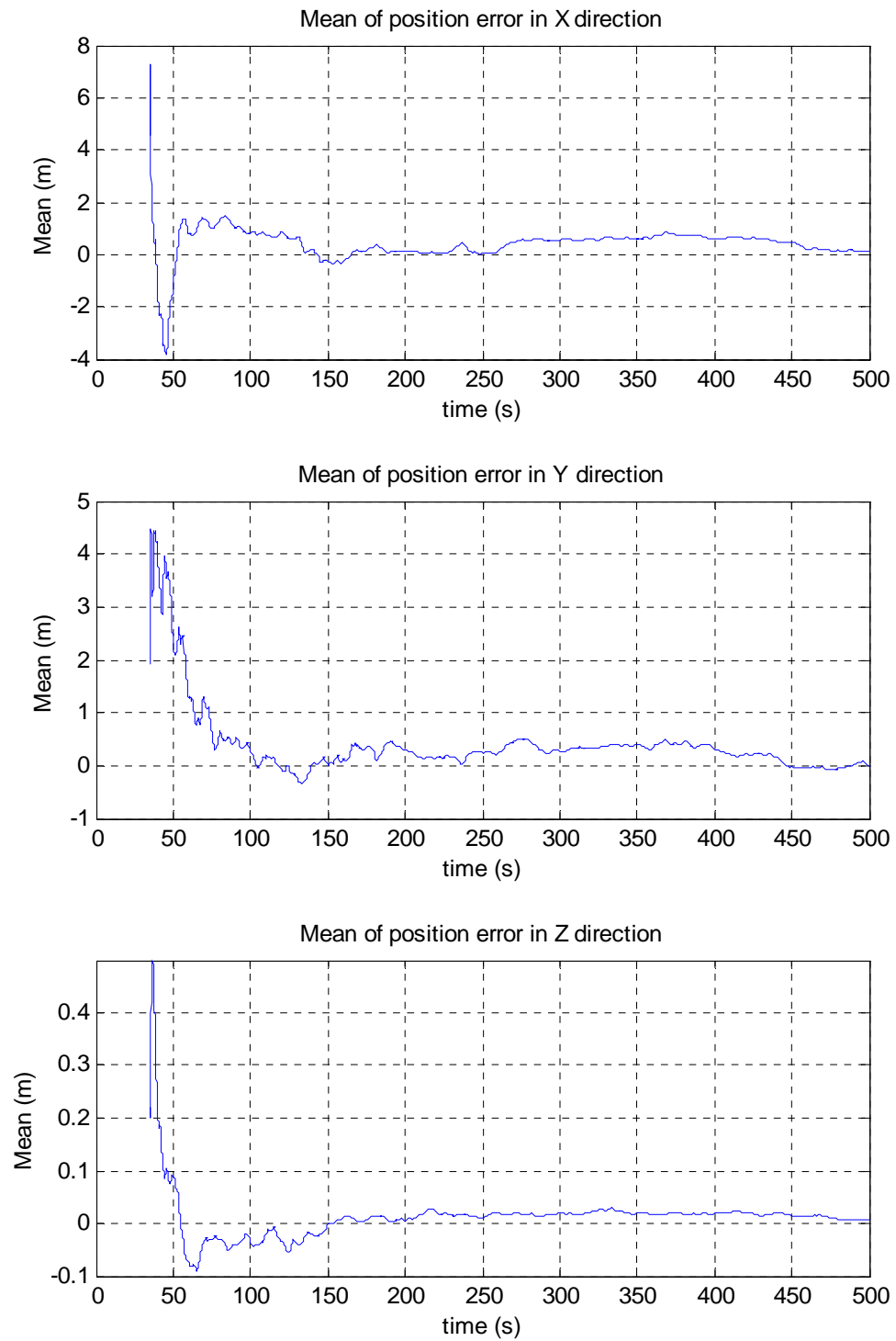


Figure 18. Mean of target position error.

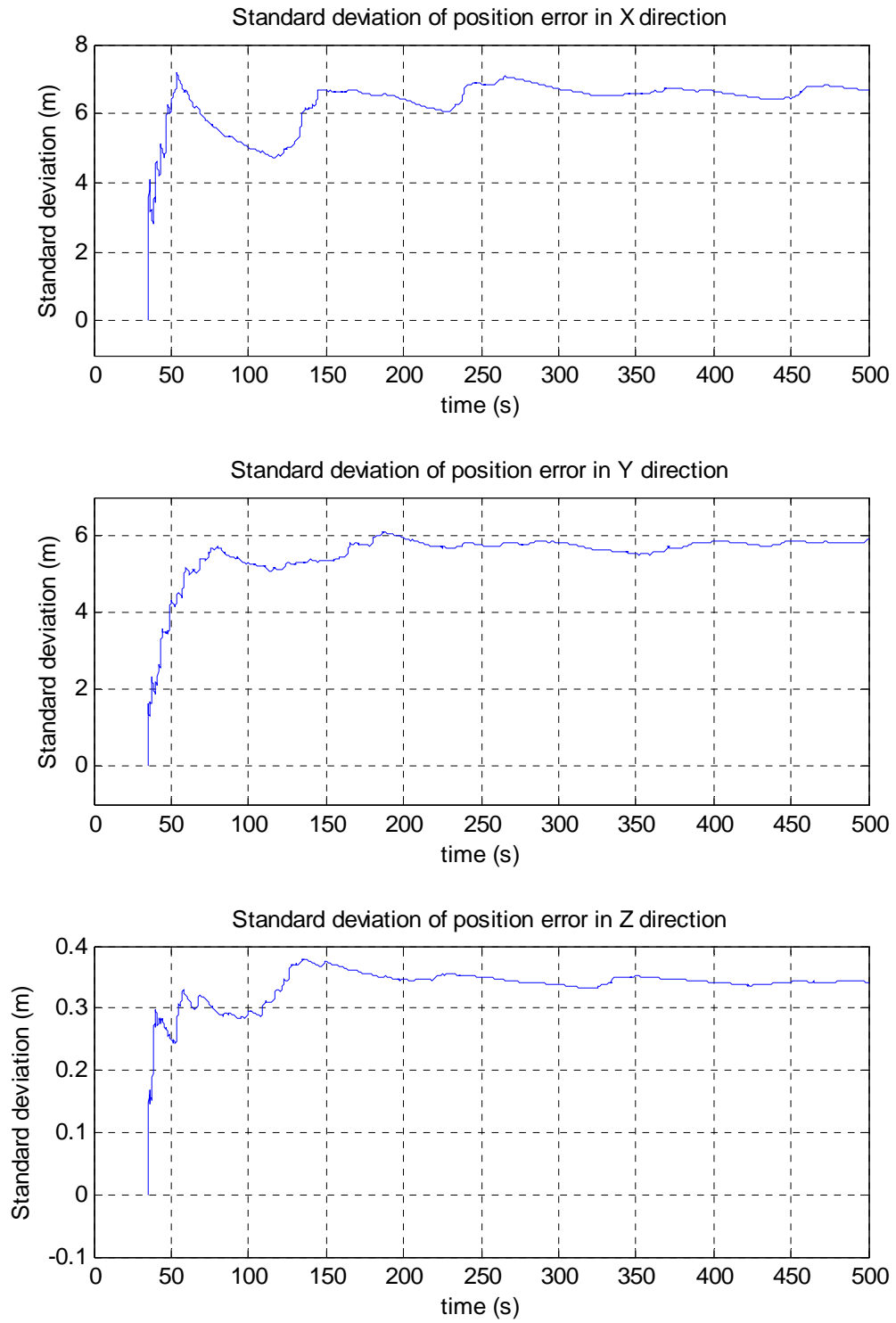


Figure 19. Standard deviation of target position error.

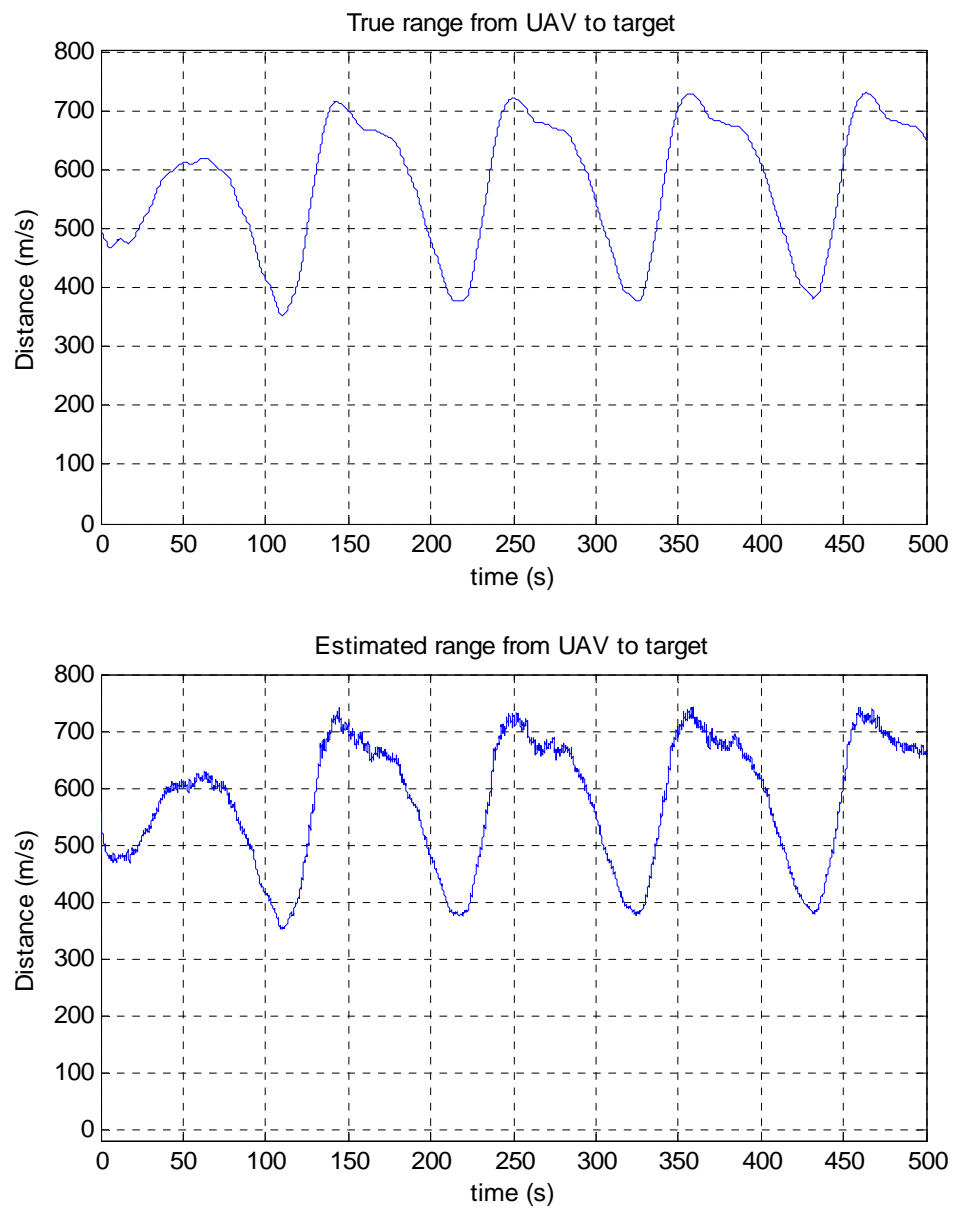


Figure 20. Range from SUAV to target (with measurement noise).

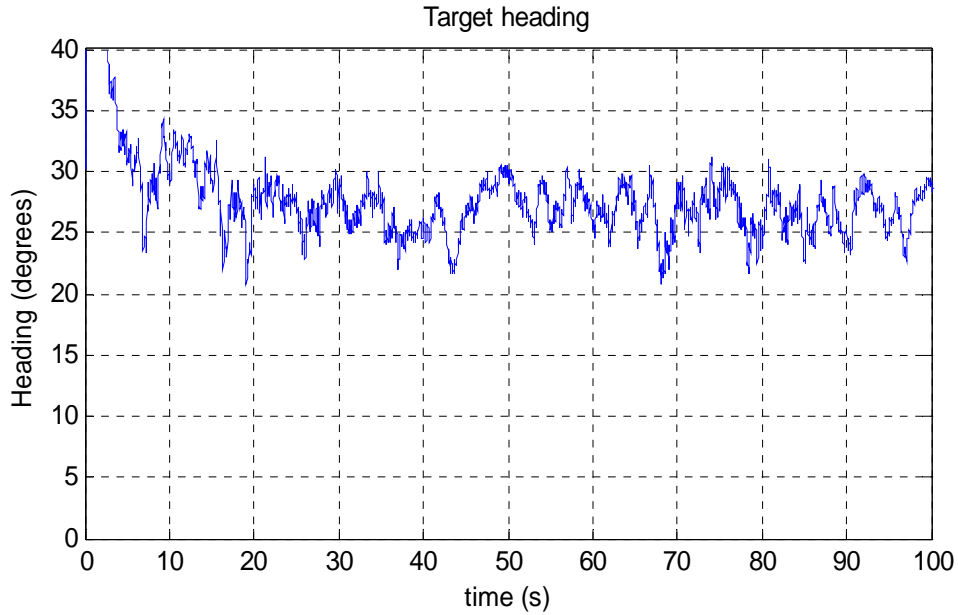


Figure 21. Estimated target heading (with measurement noise).

The convergence for the estimates was now around 25 seconds for velocity (within 90% of true velocity), based on lower gains setting by selecting the poles  $[-0.3 \ -0.3 \ -0.3 \ -0.1 \ -0.1 \ -0.1]$ . Higher gains tended to cause wider fluctuations in the estimation of target motion. Thus, reduction in gains also has the effect of reducing the fluctuations in the estimation of target motion.

In conclusion, this filter worked well in simulation in the presence of white noise. There was, however, a balance required between fast convergence time and degree of fluctuations in the target motion estimates. This can be achieved by selecting appropriate poles, hence the gains  $K_1$  and  $K_2$ .

## V. APPLICATION OF KALMAN FILTER

### A. DESCRIPTION OF FILTER

The next filter, used to estimate target motion, employed the continuous nonlinear Kalman filtering technique described in Grewal et. al. [Ref 6]. Before the filter can be implemented, the kinematics of the tracking problem must be established.

#### 1. Kinematics Equations

The following diagram showed the relations between the SUAV and moving target.

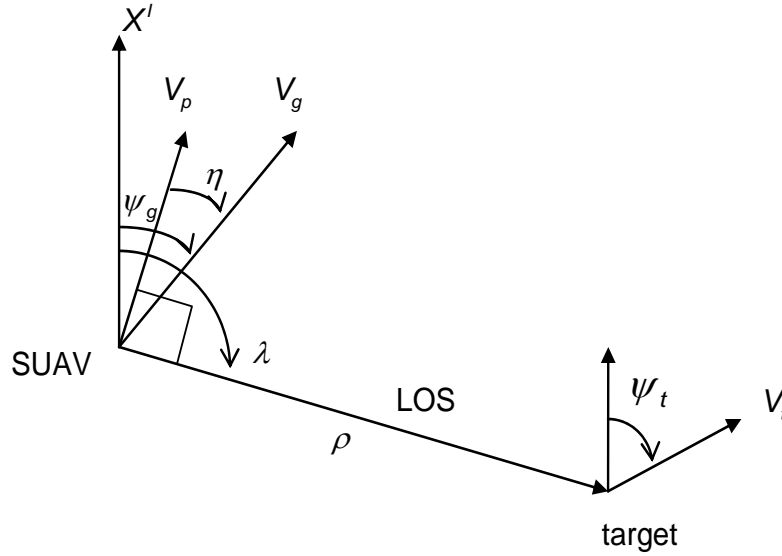


Figure 22. Target tracking in inertial frame [After: Ref 2].

The state variables  $x$  comprise the parameters  $\eta$ ,  $\rho$ ,  $\dot{\lambda}$ ,  $V_t$  and  $\psi_t$ . The target was assumed to be moving with constant velocity and heading. The resulting kinematics relationship was as follows:

$$\eta = \frac{\pi}{2} - \lambda + \psi_g \quad [\text{Ref 2}] \quad (8)$$

$$\dot{\eta} = \dot{\psi}_g - \dot{\lambda} \quad [\text{Ref 2}] \quad (9)$$

$$\dot{\rho} = -V_g \sin \eta + V_t \sin(\psi_t + \eta - \psi_g) \quad [\text{Ref 2}] \quad (10)$$

$$\dot{\lambda} = \frac{V_g \cos \eta}{\rho} - \frac{V_t \cos(\psi_t + \eta - \psi_g)}{\rho} \quad (11)$$

Letting  $\dot{V}_t = 0$ ,  $\dot{\psi}_t = 0$  and assuming  $\dot{V}_g = 0$ , we obtain

$$\ddot{\lambda} = -\frac{V_g \sin \eta}{\rho} \dot{\psi}_g + \frac{V_g \sin \eta}{\rho} \dot{\lambda} - \frac{V_g \cos \eta}{\rho^2} \dot{\rho} - \frac{V_t \sin(\psi_t + \eta - \psi_g)}{\rho} \dot{\lambda} + \frac{V_t \cos(\psi_t + \eta - \psi_g)}{\rho^2} \dot{\rho} \quad (12)$$

$$\dot{V}_t = 0 \quad (13)$$

$$\dot{\psi}_t = 0 \quad (14)$$

## 2. Process Model

The nonlinear process model was obtained from Grewal et al [Ref 6] as follows:

$$\dot{x}(t) = f(x(t), t) + w(t) \quad w(t) \sim N(0, Q(t)) \quad (15)$$

$$z(t) = h(x(t), t) + v(t) \quad v(t) \sim N(0, R(t)) \quad (16)$$

The implementation equations [Ref 6] were:

$$\dot{\hat{x}}(t) = f(\hat{x}(t), t) + \bar{K}(t)[z(t) - \hat{z}(t)] \quad (17)$$

$$\hat{z}(t) = h(\hat{x}(t), t) \quad (18)$$

The linear approximation equations [Ref 6] were:

$$F(t) \approx \left. \frac{\partial f(x, t)}{\partial x} \right|_{x=\hat{x}(t)} \quad (19)$$

$$H(t) \approx \left. \frac{\partial h(x, t)}{\partial x} \right|_{x=\hat{x}(t)} \quad (20)$$

The Kalman gain equations [Ref 6] were:

$$\dot{P}(t) = F(t)P(t) + P(t)F^T(t) + G(t)Q(t)G^T(t) - \bar{K}(t)R(t)\bar{K}^T(t) \quad (21)$$

$$\bar{K}(t) = P(t)H^T(t)R^{-1}(t) \quad (22)$$

### 3. Kalman Gain Computation

$F(t)$  was computed based on the assumption of the following constant values:  $V_g = 20$  m/s,  $\eta = 0$ , range  $\rho = 500$  m,  $\dot{\lambda} = \frac{V_g}{\rho} = 0.04$  rad/s,  $V_t = 5$  m/s,  $\dot{\psi}_g = 0.04$  rad/s.  $\psi_g$  took on the latest value from the SUAV truth model as the SUAV changes heading.  $\psi_t$  was unknown and hence took on the value from latest estimated target heading  $\hat{\psi}_t$ .

In the measurement equation (16),  $h(t)$  comprised measurements  $[\eta \ V_g]$  where  $V_g \approx \rho \dot{\lambda}$ .

To obtain steady state gain, equation (21) was set to zero, i.e., letting  $\dot{P}(t) = 0$ . The gain from equation (22) was finally obtained by solving Algebraic Riccati Equation [Ref 2] for  $P$ .

The process noise covariance  $Q$  and measurement noise covariance  $R$  were chosen as follows:

$$Q = \begin{bmatrix} 0.0001 & 0 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0.0101 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.0005 & 0 \\ 0 & 0.025 \end{bmatrix}$$

Using nominal values,  $V_g = 20$  m/s,  $\eta = 0$ , range  $\rho = 500$  m,  $\dot{\lambda} = \frac{V_g}{\rho} = 0.04$  rad/s,  $V_t = 5$  m/s and  $\dot{\psi}_g = 0.04$  rad/s, the heading difference between the SUAV and target was varied over a cycle of 360 degrees. With these inputs to  $F(t)$ , the steady state Kalman gain  $\bar{K}$  was computed as described above and its values are shown below.

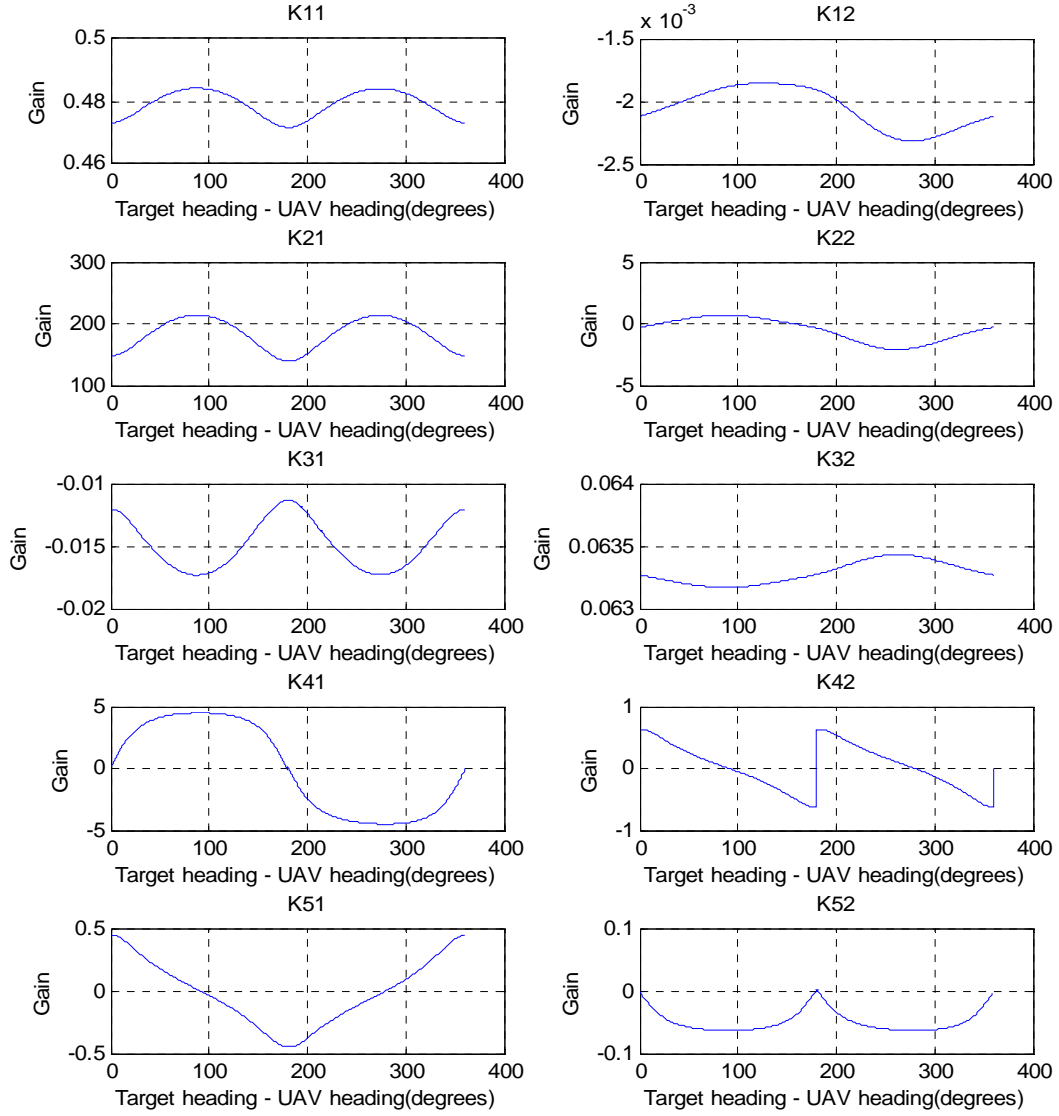


Figure 23. Kalman gain.

In the above figure, the gain was denoted by  $K_{ij}$  where the subscript  $i = 1$  to 5 was associated with the respective state variables  $\eta$ ,  $\rho$ ,  $\dot{\lambda}$ ,  $V_t$  and  $\psi_t$  in that order. The subscript  $j = 1$  to 2 was associated with the measurements  $\eta$  and  $V_g \approx \rho\dot{\lambda}$  respectively. Clearly, the gain varied according to the difference in



heading between SUAV and target. This set of gains was used to provide estimates of the state variables.

## B. RESULTS AND DISCUSSION

Equation (17) was implemented using existing simplified UAV truth model based on only one body turn rate  $\dot{\psi}_g$  in yaw, for the airframe. The filter performance is shown in figures below.

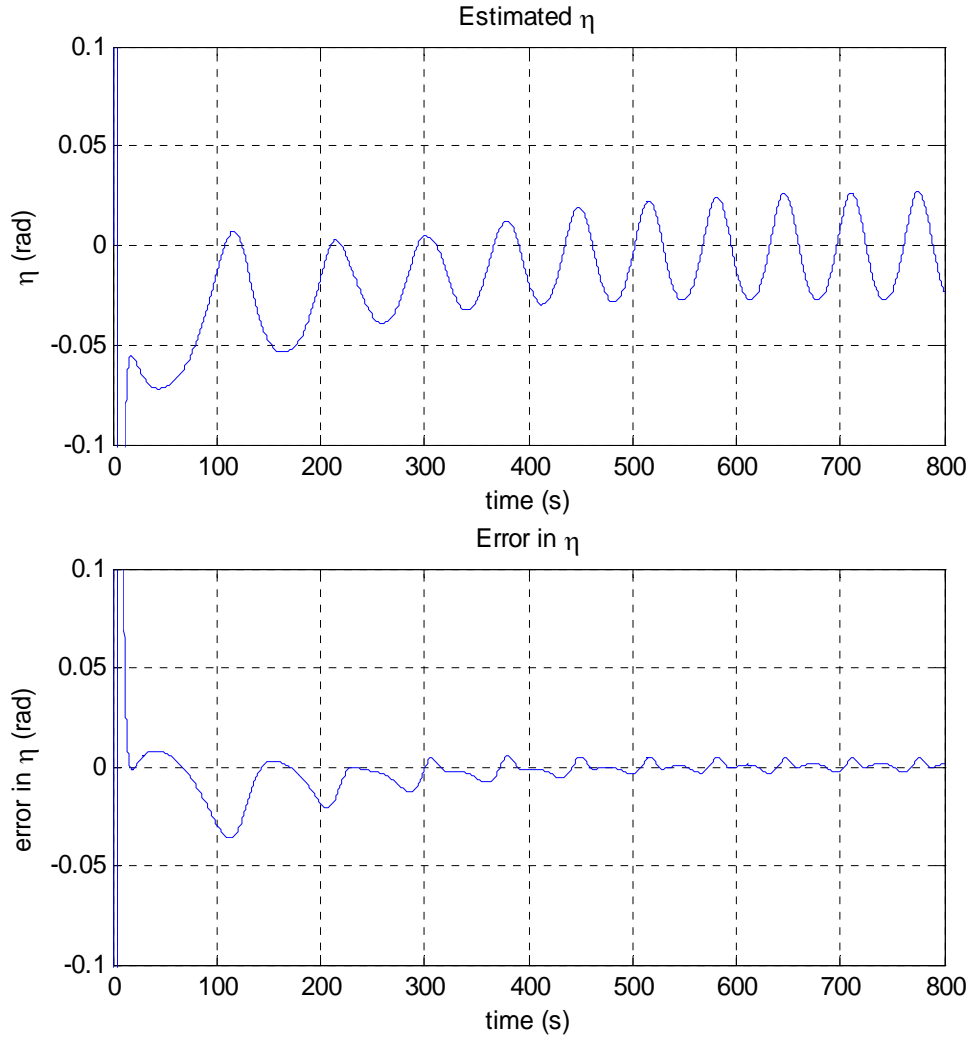


Figure 24. Estimated  $\eta$ .

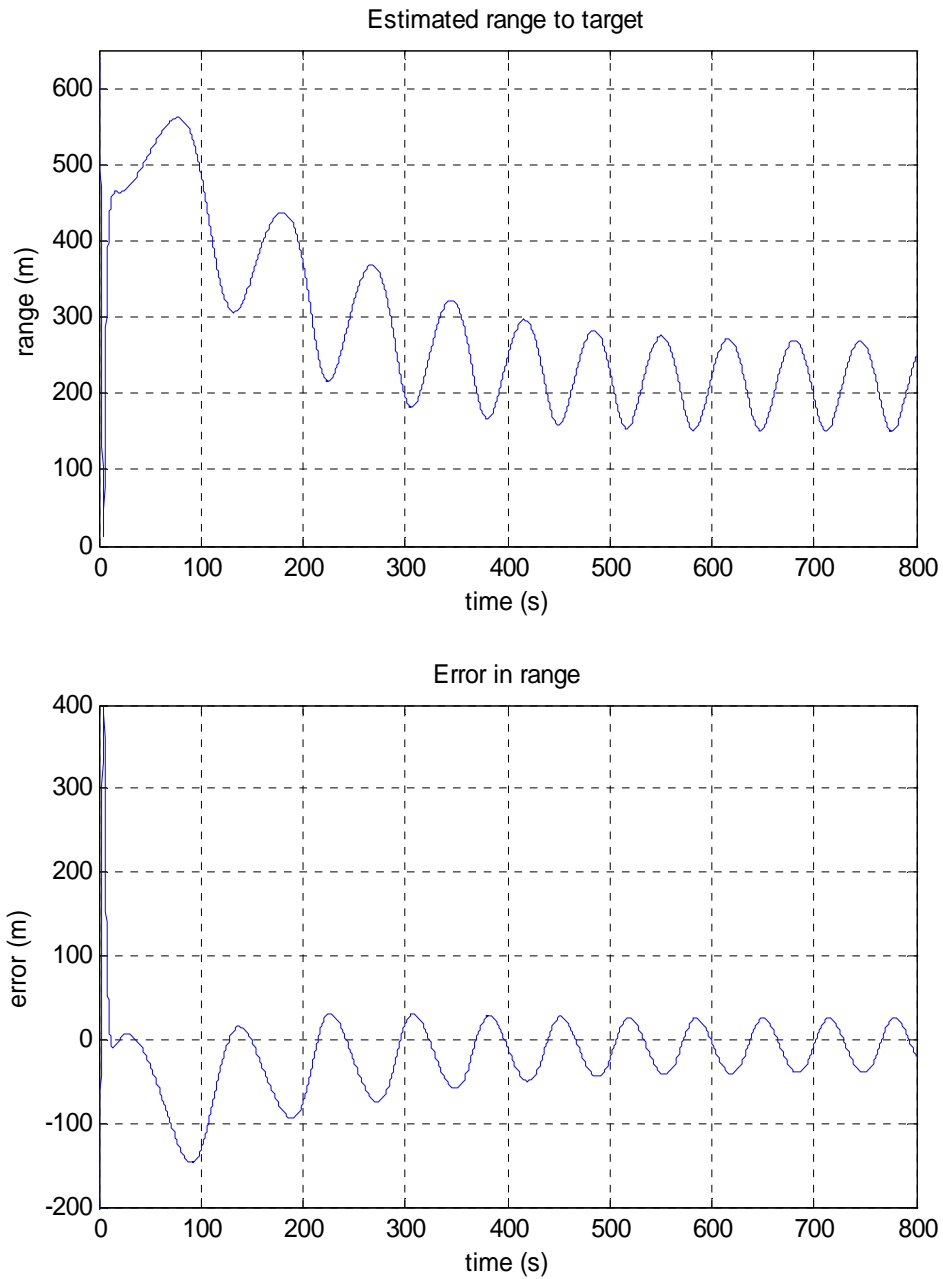


Figure 25. Estimated range from SUAV to target.

The error was about 40m (after 500 seconds) compared to the true range of approximately 200m mean, meaning an error of about 20%.

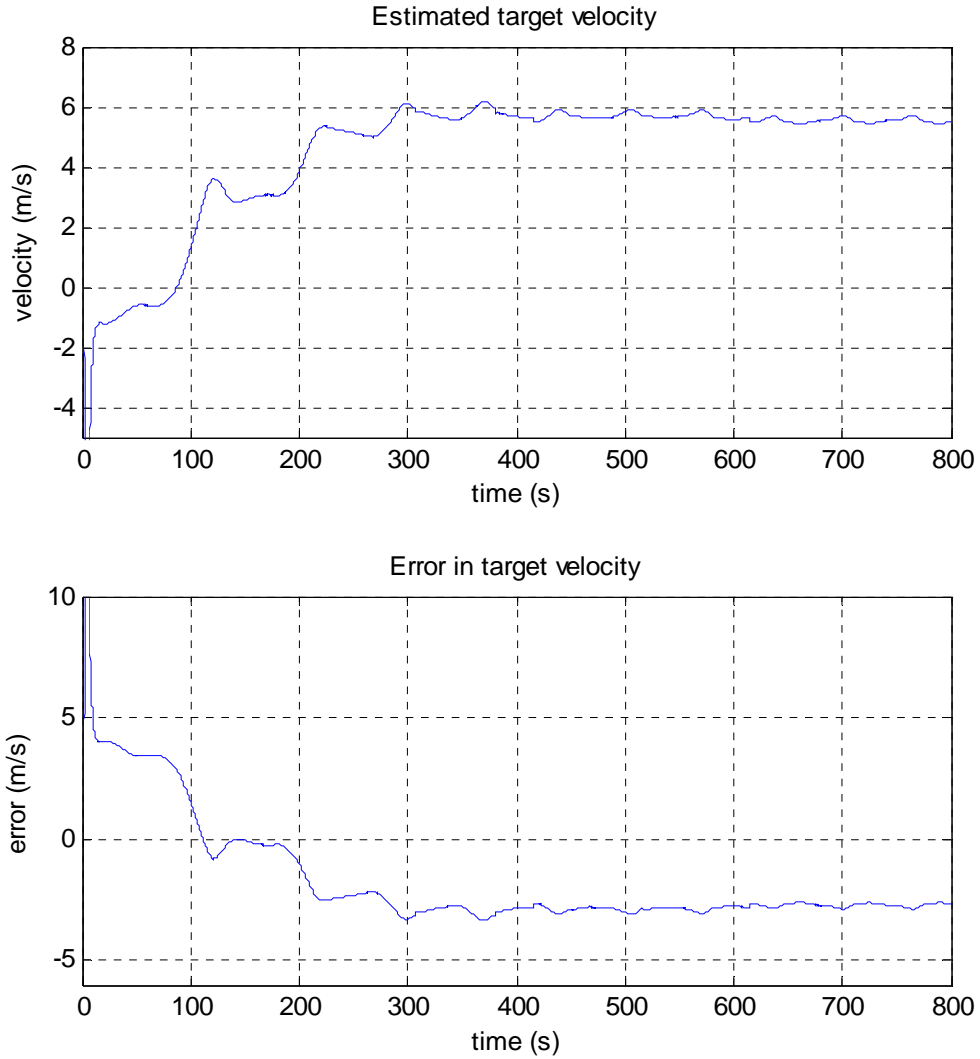


Figure 26. Estimated target velocity  $V_t$ .

The target velocity estimate was not accurate enough. It was about twice the true target velocity based on a target velocity of  $[2, 2, 0]$  m/s in the inertial frame along  $x_I$ ,  $y_I$  and  $z_I$  axis, i.e., 2.83 m/s. at a heading of 45 degrees or 0.785 rad. The large discrepancy between the true and estimated velocity could not be successfully remedied. Possible causes could be due to the choice of nominal values assumed for variables in  $H(t)$  and the low values of the Kalman gain, for target velocity, which was related to the choice of noise covariance.

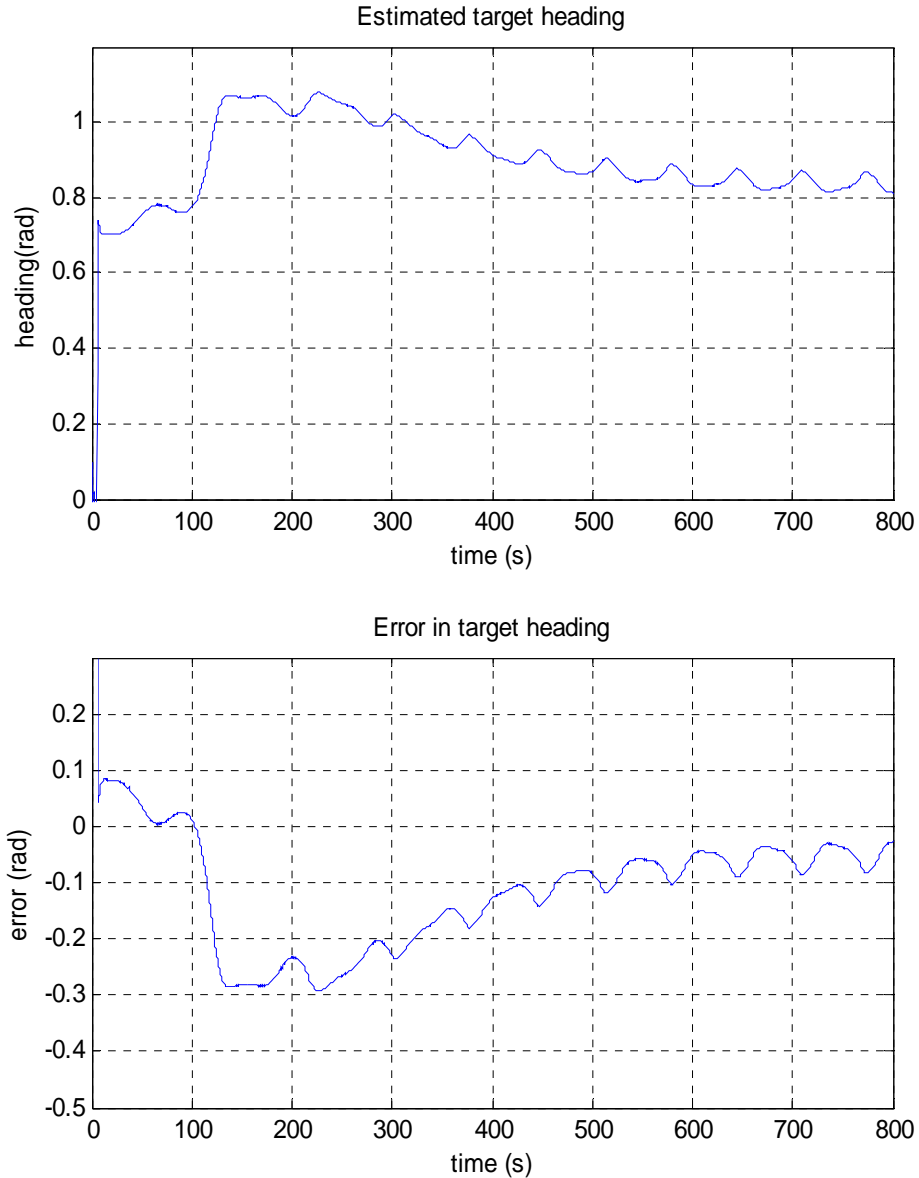


Figure 27. Estimated target heading  $\psi_t$ .

The mean heading error was about 0.06 rad average compared with the true heading of 0.785 rad.

Overall, the filter in this particular implementation could provide estimates of the state variables to within 20% error approximately except for the target velocity. In future, further assessment using Kalman filter technique will be beneficial in identifying the cause of the estimation discrepancies observed above.

## **VI. CONCLUSION AND RECOMMENDATIONS**

In the tracking of stationary ground targets, previous work by various authors was discussed. With the background gathered from both the stationary and moving targets tracking, this project attempted to apply known filtering techniques to estimation of ground target range and velocity. In this thesis, the problem of tracking moving ground target using a camera mounted on a SUAV was assessed in simulation, using two different filters.

The first filter was a nonlinear deterministic filter with stability guarantee. This technique was found to estimate the target motion with fast convergence in the region of five seconds, in the absence of measurement noise. With white noise in the measurement, the convergence time was around 25 seconds, using the gain described in this thesis. The estimates from this filter compared very well with the true target motion.

The second filter technique assessed was based on the continuous nonlinear Kalman filter with steady state gain. This approach could not estimate closely the true target motion, for the case of this particular project, when compared with the first filter technique mentioned above.

Overall, the attempt to estimate the moving ground target motion was successful using the first filtering technique. Future work is still required to verify the suitability of this filter using real flight test data.

It is recommended for future work, that actual SUAV flight and target tracking data be used to verify the effectiveness of the nonlinear deterministic filter with stability guarantee. It is further recommended that the Kalman filtering technique be studied further to explore the issues, surrounding estimation of target motion, encountered in this project.

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